all-encompassing and unavoidable form. Conscious experience confronts us with a variant of the same problem that we face with respect to function, meaning, or value. None of these phenomena are materially present either and yet they matter, so to speak. In each of these cases, there is something present that marks this curious intrinsic relation to something absent. In the case of consciousness, what is present is an awake, functioning brain, buzzing with trillions of signaling processes each second. But there is an additional issue with consciousness that makes it particularly insistent, in a way that these other absential relations aren't: *that which is explicitly absent is me.*

CALCULATING WITH ABSENCE

The difficulty we face when dealing with absences that matter has a striking historical parallel: the problems posed by the concept of zero. As the epigraph for this chapter proclaims, one of the greatest advances in the history of mathematics was the discovery of zero. A symbol designating the lack of quantity was not merely important because of the convenience it offered for notating large quantities. It transformed the very concept of number and revolutionized the process of calculation. In many ways, the discovery of the usefulness of zero marks the dawn of modern mathematics. But as many historians have noted, zero was at times feared, banned, shunned, and worshiped during the millennia-long history that preceded its acceptance in the West. And despite the fact that it is a cornerstone of mathematics and a critical building block of modern science, it remains problematic, as every child studying the operation of division soon learns.

A convention for marking the absence of numerical value was a late development in the number systems of the world. It appears to have originated as a way of notating the state of an abacus[4] when a given line of beads is left unmoved in a computation. But it literally took millennia for marking the null value to become a regular part of mathematics in the West. When it did, everything changed. Suddenly, representing very large numbers no longer required coming up with new symbols or writing unwieldy long strings of symbols. Regular procedures, algorithms, could be devised for adding, subtracting, multiplying, and dividing. Quantity could be understood in both positive and negative terms, thus defining a number line. Equations could represent geometric objects and vice versa—and much more. After centuries of denying the legitimacy of the concept—assuming that to incorporate it into reasoning about things would be a corrupting influence, and seeing its contrary properties as reasons for excluding it from quantitative analysis—European scholars eventually realized that these notions were unfortunate prejudices. In many respects, zero can be thought of as the midwife of modern science. Until Western scholars were able to make sense of the systematic properties of this non-quantity, understanding many of the most common properties of the physical world remained beyond their reach.

What zero shares in common with living and mental phenomena is that these natural processes also each owe their most fundamental character to what is specifically not present. They are also, in effect, the physical tokens of this absence. Functions and meanings are explicitly entangled with something that is not intrinsic to the artifacts or signs that constitute them. Experiences and values seem to inhere in physical relationships but are not there at the same time. This something-not-there permeates and organizes what is physically present in these phenomena. Its absent mode of existence, so to speak, is at most only a potentiality, a placeholder.

Zero is the paradigm exemplar of such a placeholder. It marks the columnar position where the quantities 1 through 9 can potentially be inserted in the recursive pattern that is our common decimal notation (e.g., the tens, hundreds, thousands columns), but it itself does not signify a quantity. Analogously, the hemoglobin molecules in my blood are also placeholders for something they are not: oxygen. Hemoglobin is exquisitely shaped in the negative image of this molecule's properties, like a mold in clay, and at the same time reflects the demands of the living system that gives rise to it. It only holds the oxygen molecule tightly enough to carry it through the circulation, where it gives it up to other tissues. It exists and exhibits these properties because it mediates a relationship between oxygen and the metabolism of an animal body. Similarly, a written word is also a placeholder. It is a pointer to a space in a network of meanings, each also pointing to one another and to potential features of the world. But a meaning is something virtual and potential. Though a meaning is more familiar to us than a hemoglobin molecule, the scientific account of concepts like function and meaning essentially lags centuries behind the sciences of these more tangible phenomena. We are, in this respect, a bit like our medieval forbears, who were quite familiar with the concepts of absence, emptiness, and so on, but could not imagine how the representation of absence could be incorporated into operations involving the quantities of things present. We take meanings and purposes for granted in our everyday lives, and yet we have been unable to incorporate these into the framework of the natural sciences. We seem only willing to admit that which is materially present into the sciences of things living and mental.

For medieval mathematicians, zero was the devil's number. The unnatural way it behaved with respect to other numbers when incorporated into calculations suggested that it could be dangerous. Even today schoolchildren are warned of the dangers of dividing by zero. Do this and you can show that 1 = 2 or that all numbers are equal.[5] In contemporary neuroscience, molecular biology, and dynamical systems theory approaches to life and mind, there is an analogous assumption about concepts like representation and purposiveness. Many of the most respected researchers in these fields have decided that these concepts are not even helpful heuristics. It is not uncommon to hear quite explicit injunctions against their use to describe organism properties or cognitive operations. The almost universal assumption is that modern computational and dynamical approaches to these subjects have made these concepts as anachronistic as phlogiston.[6]

So the idea of allowing the potentially achievable consequence characterizing a function, a reference, or an intended goal to play a causal role in our explanations of physical change has become anathema for science. A potential purpose or meaning must either be reducible to a merely physical parameter identified within the phenomenon in question, or else it must be treated as a useful fiction only allowed into discussion as a shorthand appeal to folk psychology for the sake of non-technical communication. Centuries of battling against explanations based on superstition, magic, supernatural beings, and divine purpose have trained us to be highly suspicious of any mention of such *intentional* and *teleological* properties, where things are explained as existing "for-the-sake-of" something else. These phenomena can't be what they seem. Besides, assuming that they are what they seem will almost certainly lead to absurdities as problematic as dividing by zero.

Nevertheless, learning how to operate with zero, despite the fact that it violated principles that hold for all other numbers, opened up a vast new repertoire of analytic possibilities. Mysteries that seemed logically necessary and yet obviously false not only became tractable but provided hints leading to powerful and currently indispensable tools of scientific analysis: in other words, calculus.

Consider the famous Zeno's paradox, which was framed in terms of a race between swift Achilles and a tortoise, which was given a slight head start. Zeno argued that moving any distance involved moving through an infinite series of fractions of that distance (1/2, 1/4, 1/8, 1/16 of the distance, and so on). Because of the infinite number of these fractions, Achilles could apparently never traverse them all and so would never reach the finish line. Worse yet, it appeared that Achilles could never even overtake the tortoise, because every time he reached that fraction of the distance to where the tortoise had just been, the tortoise would have moved just a bit further.

To resolve this paradox, mathematicians had to figure out how to deal with infinitely many divisions of space and time and infinitely small distances and durations. The link with calculus is that differentiation and integration (the two basic operations of calculus) represent and exploit the fact that many infinite series of mathematical operations converge to a finite solution. This is the case with Zeno's problem. Thus, running at constant speed, Achilles might cover half the distance to the finish line in 20 seconds, then the next quarter of the distance in 10 seconds, then the next smaller fraction of the distance in a correspondingly shorter span of time, and so forth, with each microscopically smaller fraction of the distance taking smaller and smaller fractions of a second to cover. The result is that the total distance can still be covered in a finite time. Taking this convergent feature into account, the operation of differentiation used in calculus allows us to measure instantaneous velocities, accelerations, and so forth, even though effectively the distance traveled in that instant is zero.

A ZENO'S PARADOX OF THE MIND

I believe that we have been under the spell of a sort of Zeno's paradox of the mind. Like the ancient mathematicians confused by the behavior of zero, and unwilling to countenance incorporating it into their calculations, we seem baffled by the fact that absent referents, unrealized ends, and abstract values have definite physical consequences, despite their apparently null physicality. As a result, we have excluded these relations from playing constitutive