Applied Calculus for AI

Duration: 3 days (24 hours)

Note: The course is heavily focused on problem-solving, with minimal theory review. Each module is designed to provide practical application scenarios in AI.

Course Objectives

This course will empower participants with the mathematical tools necessary to tackle complex challenges in AI, ensuring they can apply calculus concepts confidently and effectively in their work.

Course Outcomes

By the end of this course, participants will be able to:

1. **Apply Calculus in AI Contexts:**
   - Understand and utilize key calculus concepts such as derivatives, integrals, and multivariable calculus in the development and optimization of AI models.
2. **Optimize Machine Learning Models:**
   - Apply differentiation techniques to optimize cost functions and understand gradient descent, backpropagation, and other key algorithms in AI.
3. **Solve AI-Related Problems Using Integrals:**
   - Employ integration techniques to solve problems related to probability distributions, expectation, variance, and cumulative functions in AI.
4. **Analyze Multivariable Functions:**
   - Work with multivariable calculus to compute gradients, divergences, and solve multiple integrals, enhancing the understanding and application of complex AI models.
5. **Utilize Advanced Calculus Concepts:**
   - Apply higher-order derivatives, Taylor series, and differential equations to model time-dependent systems and dynamic behaviors in AI.
6. **Bridge Theory and Practice:**
   - Translate theoretical calculus concepts into practical problem-solving skills directly applicable to AI tasks, fostering a deeper understanding of how calculus underpins AI technologies.

Day 1: Fundamentals of Calculus and Derivatives

Module 1: Quick Review of Key Concepts (2 hours)

- **Limits and Continuity**
  - Brief review of limits, continuity, and their significance in calculus.
  - Practice problems on evaluating limits.
- **Differentiation Basics**
  - Introduction to derivatives, the power rule, product rule, quotient rule, and chain rule.
  - Solving basic differentiation problems.

Module 2: Applications of Derivatives in AI (3 hours)

- **Optimization in Machine Learning**
  - Concept of gradient and its role in optimization.
  - Practice problems on finding local maxima and minima.
  - Gradient Descent: Understanding the algorithm through problem-solving.
- **Backpropagation in Neural Networks**
  - Role of calculus in backpropagation.
  - Practical problems related to partial derivatives and their applications in adjusting weights.

Module 3: Higher-Order Derivatives (3 hours)

- **Second and Higher-Order Derivatives**
  - Understanding concavity, convexity, and points of inflection.
  - Practice problems on concavity, convexity, and their relevance in machine learning models.
- **Taylor Series Expansion**
  - Application in approximating functions and understanding model behaviour.
  - Problems focused on series expansion in the context of AI algorithms.

Day 2: Integrals and Their Applications

Module 4: Basics of Integration (2 hours)

- **Indefinite and Definite Integrals**

    - Introduction to integration techniques: substitution, integration by parts.
    - Practice problems focusing on indefinite and definite integrals.
- **Application of Integration in AI**
    - Concept of area under the curve and its relevance in probability distributions and cost functions.

Module 5: Probability and Statistics in AI (3 hours)

- **Cumulative Distribution Function (CDF) and Probability Density Function (PDF)**
    - Role of integration in understanding probability distributions.
    - Practice problems focused on calculating areas under PDFs and working with CDFs.
- **Expectation and Variance**
    - Calculating expectation and variance using integrals.
    - Practical problems on computing moments and their application in statistical models.

Module 6: Multivariable Calculus and AI Applications (3 hours)

- **Multiple Integrals**
    - Double and triple integrals, with a focus on their applications in AI (e.g., volume under surfaces, joint probability distributions).
    - Practice problems involving multiple integrals and their application in AI algorithms.
- **Gradient, Divergence, and Curl**
    - Introduction to vector calculus and its relevance in machine learning.
    - Practical problems to solidify understanding of these concepts.

Day 3: Advanced Calculus Topics and AI Integration (Optional)

Module 7: Differential Equations and Dynamical Systems (4 hours)

- **Ordinary Differential Equations (ODEs)**
    - Introduction to ODEs and their role in AI (e.g., in modelling time-dependent systems).
    - Practice problems focused on solving ODEs using separation of variables and integrating factors.

- **Partial Differential Equations (PDEs)**
    - Introduction to PDEs with examples from AI applications (e.g., heat equation in neural networks).
    - Solving basic PDEs using separation of variables and Fourier series.

Module 8: Calculus of Variations and AI (4 hours)

- **Introduction to Calculus of Variations**
    - Application in optimizing functional, relevant in deep learning.
    - Practice problems on Euler-Lagrange equations.
- **Application in AI and Deep Learning**
    - Understanding the variational approach in AI models.
    - Solving problems that link calculus of variations with AI, such as deriving equations used in advanced AI models.
